

Clocks & Time: 8 Important Shortcuts Explained

We need to get couple of basic facts clear:

* Speed of the hour hand = 0.5 degrees per minute (dpm) {The hour hand completes a full circle or 360 degrees in 12 hours or 720 minutes}

* Speed of the minute hand = 6 dpm {The minute hand completes a full circle in 60 minutes}

* At 'n' o' clock, the angle of the hour hand from the vertical is $30n$

Clock problems can be broadly classified in two categories:

a) Problems on angles

b) Problems on incorrect clocks

c) Problems on angles

Finding the angle between the hands category clock problems major questions which is quite time taking and difficult to solve.

Some easy tricks to solve

Trick-1:

The time is X h : Y min. so, the angle between hour and minute hand is given by

Angle between X and Y

$$= |(X*30) - ((Y*11)/2)| \text{ or}$$

$$30 * [HRS - (MIN/5)] + (MIN/2)$$

Ex : Angle between hands at 5:30 ?

a. 15 b. 20 c. 25 d. 30

Sol : Angle between 5 and 30 = $|(5*30) - ((30*11)/2)|$

$$= |150 - 165|$$

$$= 15 \text{ degrees}$$

Thus, angle between hands at 5:30 is 15 degrees.

Trick-2:

Right angles at each other by hands of clock between one hour is given by

$$= |(5x \pm 15) * (12/11)|$$

Ex : At what time do the hands of a clock between 7:00 and 8:00 form 90 degrees?

Sol: there is two possibles

Case-1:

$$= (5*7 - 15) * (12/11)$$

$$= 240/11$$

$$= 21 \text{ minutes } 9/11 \text{ of a minute}$$

Case-2:

$$= (5 \cdot 7 + 15) \cdot (12/11)$$

$$= 600/11$$

$$= 54 \text{ minutes } 6/11 \text{ of a minute}$$

So, the hands of the clock are at 90 degrees at the following timings:

$$7 : 21 : 9/11 \text{ and } 7 : 54 : 6/11$$

Trick-3:

For coinciding the hands between one hour is

$$= (5x) \cdot (12/11)$$

Ex: At what time do the hands of the clock meet between 7:00 and 8:00 ?

$$\text{Sol: } = (60 \cdot 7)/11$$

$$= 420/11$$

$$= 38 \text{ minutes } 2/11 \text{ minute}$$

Hands of the clock meet at

$$7 : 38 : 2/11$$

Opposite Direction , $(5x - 30) \cdot (12/11)$

Trick-4:

Opposite to each other by hands of clock between one hour is given by

$$= | (5x - 30) \cdot (12/11) |$$

Ex : At what time do the hands of a clock between 7:00 and 8:00 form 180 degrees?

Sol:

$$= | (5 \cdot 7 - 30) \cdot (12/11) |$$

$$= 60/11$$

$$= 5 \text{ minutes } 5/11 \text{ of a minute}$$

Trick-5

Some other results which might be useful:

· Hands of a clock meet at a gap of $65 \frac{5}{11}$ minutes.

The meetings take place at 12:00:00, 1:05:5/11, 2:10:10/11 ... and so on.

· A clock makes two right angles between 2 hours. The clock does not make 48 right angles in 24 hours (1 day)

· Hands of a clock meet 11 times in 12 hours and 22 times in a day.

· Hands of a clock are perfectly opposite to each other (i.e. 180 degrees) 11 times in 12 hours and 22 times a day. {Same as above}

· Any other angle is made 22 times in 12 hours and 44 times in a day

Number of right angles

Note: Between 2 -4 and 8-10 there are 3 right angles and not 4. The first right angle between 3 - 4 (9-10) and second right angle between 2-3(8-9) are the same.

Problems on incorrect clocks

Such sort of problems arise when a clock runs faster or slower than expected pace. When solving these problems it is best to keep track of the correct clock.

Type-1:

Ex: A watch gains 5 seconds in 3 minutes and was set right at 8 AM. What time will it show at 10 PM on the same day?

Ans: The watch gains 5 seconds in 3 minutes \Rightarrow 100 seconds in 1 hour.

From 8 AM to 10 PM on the same day, time passed is 14 hours.

In 14 hours, the watch would have gained 1400 seconds or 23 minutes 20 seconds.

So, when the correct time is 10 PM, the watch would show 10 : 23 : 20 PM

Type-2:

Ex: A watch gains 5 seconds in 3 minutes and was set right at 8 AM. If it shows 5:15 in the afternoon on the same day, what is the correct time?

Sol: The watch gains 5 seconds in 3 minutes \Rightarrow 1 minute in 36 minutes

From 8 AM to 5:15, the incorrect watch has moved 9 hours and 15 minutes = 555 minutes.

When the incorrect watch moves for 37 minutes, correct watch moves for 36 minutes.

\Rightarrow When the incorrect watch moves for 1 minute, correct watch moves for $\frac{36}{37}$ minutes

\Rightarrow When the incorrect watch moves for 555 minutes, correct watch moves for $(\frac{36}{37}) * 555 = 36 * 15$ minutes = 9 hours

\Rightarrow 9 hours from 8 AM is 5 PM.

\Rightarrow The correct time is 5 PM.

Note: I am sure you would have heard the proverb that even a broken clock is right twice a day. However, a clock which gains or loses a few minutes might not be right twice a day or even once a day. It would be right when it had gained / lost exactly 12 hours.

Type-3:

Ex: A watch loses 5 minutes every hour and was set right at 8 AM on a Monday. When will it show the correct time again?

Ans: For the watch to show the correct time again, it should lose 12 hours.

It loses 5 minutes in 1 hour

=> It loses 1 minute in 12 minutes

=> It will lose 12 hours (or 720 minutes) in 720×12 minutes = 144 hours = 6 days

=> It will show the correct time again at 8 AM on Sunday.